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Quantization Analogy of Classical and Quantum Worlds as a Teaching Approach

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ABSTRACT

In contrast to the determinist and non-discrete structure of classical physical concepts, the probabilistic and discrete/quantised structure of quantum physics embeds certain difficulties in deeper understanding and teaching activities. In this study, in order to understand and to teach the concept of quantization more effectively, a clear analogy has been set between the classical world, that is well-known modes on a rope and the quantum world, that is the quantization in the infinite square quantum well. The established analogy is exceptionally beneficial in order to internalise and teach the hard concept of quantization in a much better way.

Keywords: *physics education, quantization, quantum physics teaching, analogy teaching, analogy*

Introduction

Physics is the most fundamental branch of science that explores events occurring at all scales, involves energy and/or mass within nature, and reveals relationships between highly complicated concepts (Serway & Beichner, 2000; Halliday, Resnick & Walker, 2010). Teaching scientific laws or principles of the concepts that emerges in physics is another very important aspect of the subject. The field of physics education aims to teach the concepts and laws of physics in the most efficient and truthful way. Recent studies in the field of physics education have achieved significant progress around the globe. Some topics in physics education such as misconceptions, concept teaching, academic achievement, and active learning have come to the forefront in recent years (Marshman & Singh, 2015; Sadaghiani & Pollock, 2015; Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2017; Didiş, 2015).

Modelling and analogy are very important strategic approaches used in science and physics education. Modelling generally aims at defining and teaching a complex set of concepts and relationships with a simple and easily understood set of two-dimensional or three-dimensional materials (Koponen, 2007; Van Driel & Verloop, 1999). On the other hand, analogy proposes to teach the concepts that are more difficult to understand and teach using a set of concepts and events that are previously known and easily accessible (You, 2019; Thagard, 1992). Analogy is based on a simple and understandable model of the target concept and event. While doing this work, it is ensured that learning is realized by transferring the existing knowledge to the unknown and complex concepts and systems by emphasizing the concepts and events that the students know well. It is very important that the model takes into account an analogy that is capable of and deep enough to explain the relationship between concepts and target concepts (Körhasan & Hıdır, 2019; Clement, 1993). If there is no such qualification, serious problems or important misconceptions may emerge when teaching the target concepts and principles. The instructor must be very careful and make the necessary warnings when students establish non-real relationships between the target concept and the known concept during learning. For example, the narrowing of the diameter of the pipe during the flow of water through a pipe causes the flow rate to increase but there is no such a situation when an electric current passes through a conductive wire. Therefore, the establishment of such a relationship by students represents an important scientific misconception. Thus, the instructor should determine the boundaries of the analogy very well (Podolefsky & Finkelstein, 2006).

Studies on the learning and teaching of quantum physics have recently become an area in which physics education researchers are intensely interested (Hadzidaki, Kalkanis & Stavrou, 2000; Mashhadi, 1995; Singh, 2008; Singh, Belloni & Christian, 2006; Vella, 2002; Zollman, Rebello & Hogg, 2002). It is rather clear that pedagogical studies on this subject focus on conceptual learning, visualization, mathematical thinking, and problem solving. Robblee and Abegg (1999), revealed that teaching quantum physics using computer technology increased the level of students' understandings of quantum physical concepts. Rebello and Zollman (1999), have applied visual materials they developed for high school students with the help of computers and they have revealed that the students' misconceptions substantially decreased. Ayene, Kriek & Damtie (2011), proposed an approach to determine phenomenographic category of description of tertiary physics students' illustrations, concerning the wave-particle duality and uncertainty principle. Styer (1996), identified students' misconceptions about quantum states, measurement, identical particles and other quantum concepts. Singh et al. (2006), investigated the misconceptions about the Schrödinger wave equation and suggested that these misconceptions are the result of false generalizations. Özdemir and Erol (2008) investigated how to learn concepts such as atoms, localization, and Heisenberg uncertainty on physics teacher candidates and emphasized that non-physics related students could have difficulties learning the concepts of quantum physics.

Paatz, Ryder, Schwedes & Scott (2004), carried out a case study analysing the process of analogy-based learning in a teaching unit about simple electric circuits. Analogical thinking processes are modelled by a sequence of four steps according to Gentner's structure mapping theory, namely activate base domain, postulate local matches, connect them to a global match and draw candidate inferences. Jonāne (2015a), recently studied on analogies employed in Physics education specifically reported on views and experiences of Latvian teachers'. The study analyzed usage of analogies in physics textbooks for Basic and High School in Latvia. Seven physics textbooks are examined by using the descriptive analysis method. The usage of analogies were analysed to discuss their effectiveness for a deeper acquisition of science concepts and phenomena, for developing students' reasoning, meaning making, and transfer skills during teaching physics. Jonāne, (2015b), in another study, worked and investigated the usage and effects of analogies in science education. The application of analogies in the context of sustainable education involves richer potential. The purposeful use of appropriate analogies can facilitate analogical thinking and transfer skills, as well as develop abilities which are required for life and lifelong learning, including successful integration into modern society and facility within our technology saturated world.

The present work is original in terms of setting a clear analogy between the quantum world and the classical world. It is exceptionally important to note that, there is no study on this subject and this issue is being discussed for the first time. In addition, it is thought that this study will provide a significant contribution to quantum teaching for the students who are studying physics at undergraduate level. This research aims to establish a basis for a better understanding and teaching of quantization concept by establishing an analogy between the modes of a rope with fixed ends and the well-known infinite square quantum well.

The Concept of Quantization

Quantum mechanics focuses on resolving and explaining the structure of the microscopic/atomic world. All around us are atoms, molecules and solids with unique properties that cannot be explained with classical physics, instead requiring quantum mechanics. For example, quantum mechanics can tell us why sodium lamps are yellow, why laser diodes have a unique colour and why uranium is radioactive. The key to understanding the structure of microscopic systems lies in the energy states that the systems are allowed to have. Each microscopic system has a unique set of energy levels that gives that system a 'fingerprint' that sets it apart from other systems. With the tools of quantum mechanics, we can build a theoretical model for the system, predict that fingerprint and compare it to the experimental measurement.

Quantum mechanics demonstrate many fundamental interesting and counterintuitive properties such as wave particle duality, probably and quantization. Quantization phenomena occur in the quantum world if the movement of a particle is limited. In physics 'quantization' refers to a mathematical procedure designed to describe a quantum system using its formulation as a classical system. The understanding of the meaning of quantization seems to be the main problem in understanding quantum concepts. It is generally accepted that the quantization is an algorithm by means of which a quantum system corresponds to a classical dynamic one (Berezin, 1975). The reason why the energy is quantized can be understood through the most popular model problem that is the infinite square quantum well or particle in a box. In order to fit within the box, a sinusoidal wave function must have an integer number of bumps. Partial bumps are not allowed, because the wave function must go to zero at both ends of the box. The discontinuity at either end of the wave function would be equivalent to an infinitely short wavelength at that point, and hence an infinitely high energy subverting our goal of finding wave functions for which the energy is well defined and finite (Schroeder, 2010).

Classical Resolution of Modes on a Rope with Fixed Ends

To understand and internalise the phenomena of Quantisation in an enhanced way and to overcome some teaching difficulties setting an analogy between the quantum mechanics and of course the classical mechanics would surely be very beneficial. As a traditional scientific attitude, quantisation is assumed to be occurring at only atomic or nano-scales and therefore no efforts, to our knowledge, have been spent to create a link between the macroscopic and quantum worlds. However, a very simple macroscopic system, a stretched rope with fixed two ends, obviously demonstrates quantised modes that are exactly the same with the modes of the infinite square well, the most popularly studied quantum mechanical model problem.

A stretched classical rope with a length of L, with two fixed ends, is considered and resolved in terms of the possible waves and modes. The classical wave equation is basically employed to obtain all physical properties of the motion, which is given by,

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi(x,t)}{\partial t^2} \qquad (1)$$

where Ψ(x,t) denotes the position and time dependent wave function, and v represents the velocity of the waves. To establish an analogy, one needs to resolve the problem spatially hence the spatial wave equation is employed at this stage, that is,

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} + k^2 \Psi(x) = 0 \qquad (2)$$

where k simply denotes the wave number of the classical waves on the rope. The general solution of this spatial differential equation can be expressed as,

$$\Psi(x) = A\, sinkx + B\, cos\, kx \qquad (3)$$

where A and B are the constants and basically determined by the initial and limit conditions of the case. The most obvious physical condition at this stage can be stated as when the two ends of the string are fixed then the displacement of the string at two ends must be zero. A transverse wave travelling along the string towards a fixed end will be reflected in the opposite direction and two waves travelling in the opposite direction simply bounce back and forth between the two ends. Setting the first condition that is, when x=0 then $\Psi(x) = 0$ simply leads to the result of B=0. Similarly applying the second physical condition that is when x=L then again the condition of $\Psi(x) = 0$ must be satisfied, leading to the vital and most crucial condition of,

$$k = \frac{n\,\pi}{L} \qquad (4)$$

where n= 1, 2, 3, 4 ... a positive integer. This basic condition initiates the Quantisation and indeed the possible modes occurring on the rope. The equation gives the following modes.

$$k_n = \frac{n\,\pi}{L} = \frac{2\,\pi}{\lambda_n} \qquad (5)$$

The equations above show that a rope of L-length with fixed ends has natural vibration modes. These are stable vibrations, and each point on the rope vibrates transversely to make simple harmonic motion with constant amplitude and the same vibration frequency. Such vibrations are called normal modes of the rope. The system consists of N particles on the tensioned rope. A continuous system will have theoretically infinite grain fashion. More openly the modes can be stated as follows:

$$\lambda_1 = \frac{2\,L}{1},\ \lambda_2 = \frac{2\,L}{2}\,, \lambda_3 = \frac{3\,L}{2} \qquad (6)$$

Traveling waves have high points called crests and low points called troughs (in the transverse case) or compressed points called compressions and stretched points called rarefactions (in the longitudinal case) that travel through the medium. Standing waves don't go anywhere, but they do have regions where the disturbance of the wave is quite small, almost zero. These locations are called nodes. There are also regions where the disturbance is quite intense, greater than anywhere else in the medium, called antinodes. Actual wave functions of certain modes can simply be given by,

$$\psi_n(x) = Asin(\frac{n\pi}{L}\ x) \qquad (7)$$

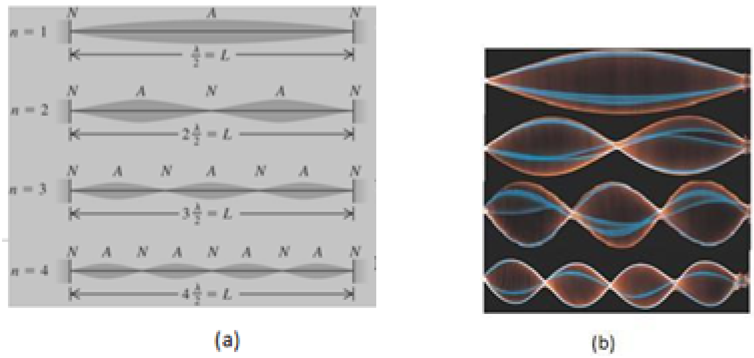

Figure 1. The shape of the first four modes on a stretched string. (a) Schematic drawing (Here is shown N node and A an antinode) and (b) experimental photographs.

It can be imagined that, on a rope with fixed ends, an infinite number of small masses are connected to each other on a continuous string. There are no vibrations at the rope's fixed ends.

Quantization of Infinite Square Quantum Well

One of the most fundamental model problems in quantum physics is the infinite square quantum well problem. The infinite potential well is a quantum system that is modelled to resolve the behaviour of a particle confined within certain limits. The case of an electron being trapped in one dimension, x, between two infinite height potential energy walls is a situation of infinite potential well. In this case, all the physical properties of the electron are found from the wave function obtained from the solution of the Schrödinger wave equation. In this specific case, we take a potential well with a width of L in the x direction and assume an electron of mass m and potential energy of V(x)=0, is trapped inside this potential well. The potential energy beyond the walls in other words outside the well is set to be infinite.

The wave functions for a confined electron with a total mechanical energy of E, mass of m and potential energy of V(x) are obtained from the solution of the time independent Schrödinger wave equation, given below,

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0 \qquad (8)$$

where, $\Psi(x)$ denotes the actual wave function accompanying the electron, and $\hbar$ is the well-known reduced Planck' constant. The solution of the equation within the condition of V(x)=0 yields exactly the identical wave functions of the classical case, that is the equation of (3). It is very important to underline that the spatial classical wave function and the time independent Schrödinger wave equation are identical with the consideration of

$$\frac{2m}{\hbar^2}[E - V] = k^2 \qquad (9)$$

Therefore, the general solution of the equation must be identical and is given by the equation (3). In this specific case, the length of the rope and the width of the quantum well are represented with the same letter that is L. Similar to the classical case, the boundary conditions determine the actual constants of A and B. The first boundary condition is, when x=0 then $\Psi(x) = 0$, which simply leads to the result of B=0 . Similarly, applying the second physical condition that is when x=L, then the wave function must be vanishing that means, $\Psi(x) = 0$. This specific condition again leads to the same vital and most crucial condition of (4) and of course (5). Therefore, the normalized Schrödinger wave functions for the electron in one dimension are given as follows:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad (10)$$

In this equation n is the quantum number and can take integer values such as 1,2,3 ... Each quantum number corresponds to a quantum state and the physical properties for that quantum state are determined by wave functions such as $\Psi_1(x)$, $\Psi_2(x)$, $\Psi_3(x)$... The first three wave functions for the infinite potential well and their graphs are given below,

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right), \psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right), \psi_3(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right) \qquad (11)$$

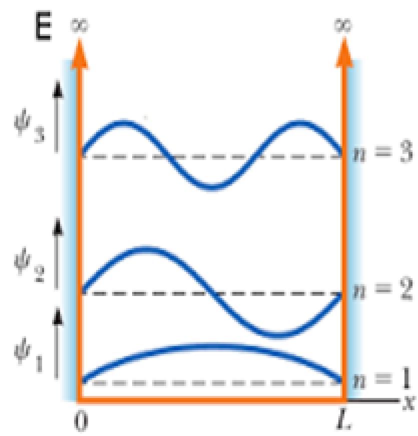

Figure 2. Wave functions of the first three wave functions accompanying the particle in the infinite potential well.

The possible wave functions can surely be expressed by means of the wave lengths as has been in the classical case. The wave numbers of course identical with the modes given in the equation (5) and wave lengths are given in the equation (6). It is quite interesting to note that the Quantisation of the wave functions in the infinite quantum well is exactly the same with the modes of the rope with two ends fixed. This situation can surely be employed to set an analogy between the two cases and would surely improve the understanding and teaching abilities and environment.

Conclusions

In this study, it is aimed to establish an analogy between the modes on rope with fixed ends in classical physics and the quantization of the infinite potential well problem in quantum physics for the better understanding and teaching of the concept of quantization in quantum physics. For this purpose, firstly resolution of the waves and modes on a macroscopic rope is carried out and specific wave functions and certain modes are extracted. Then, the problem of infinite potential well which is one of the most basic model problems in quantum physics, is fully resolved. In order to teach the concept of quantization that forms the basis of quantum physics, one of the most fundamental outcomes of classical physics is considered. As a result of the research, it has been stated that a clear and strong analogy can be achieved between the quantized states of the infinite square quantum well. It is thought that the establishment of such an analogy would provide a convenience for understanding the concept of quantization in quantum physics courses at undergraduate level. This result is supported by the relevant work carried out on quantum mechanics concept assessment (Sadaghiani & Pollock, 2015). In addition, the generated analogy was limited between the infinite potential well problem in quantum physics and the classical modes of a rope with fixed two ends. Thus, it is thought that similarities, differences, sufficient and inadequate situations should be revealed when creating analogies (Hadzidaki et al., 2000; Johnston, Crawford & Fletcher, 2008).

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